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Gravity localization on string-like defects in codimension two and the AdS/CFT correspondence

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Abstract

We study the localization of gravity on string-like defects in codimension two. We point out that the gravity-localizing 'local cosmic string' spacetime has an orbifold singularity at the horizon. The supergravity embedding and the AdS/CFT correspondence suggest ways to resolve the singularity. We find two resolutions of the singularity that have a semiclassical gravity description and study their effect on the low-energy physics on the defect. The first resolution leads, at long distances, to a codimension one Randall-Sundrum scenario. In the second case, the infrared physics is like that of a conventional finite-size Kaluza-Klein compactification, with no power-law corrections to the gravitational potential. Similar resolutions apply also in higher codimension gravity-localizing backgrounds.

1 Introduction

The idea that gravity can be localized on a domain wall in a space of infinite transverse size [1] has been the subject of much recent interest, both from phenomenological and theoretical points of view. In particular, it has been argued that the Randall-Sundrum (RS) scenario can be given a dual four-dimensional description [2] via the AdS/CFT correspondence [3, 4, 5]. In this description, the four dimensional observable matter is coupled to a four dimensional "hidden" CFT (strongly coupled, large-N) via gravity only (to define this theory, an ultraviolet cutoff is needed; at energy scales sufficiently below the cutoff the details of the cutoff are unimportant). This duality has been subjected to some quantitative tests. Notably, identical power-law corrections to Newton's law are obtained, which are due to exchange of a continuum of "Kaluza-Klein" gravitons in the five dimensional semiclassical gravity description of RS, or, in the dual CFT description, to (summed-up) hidden sector loops [2].

Following the proposal of RS, several generalizations to higher codimension have been put forward. Some of these involve localization to an intersection of domain walls—each a codimension one object—in some higher dimensional space [6]; we will not study them here. A different proposal—that gravity can be localized on a "stringlike" (codimension two) defect in AdS_6 was made in ref. [7] and subsequently generalized to higher codimension in [8]. It is this proposal we focus on here.

We ask whether there is a dual description like that of the codimension one RS case and find that the answer is affirmative. Moreover, as we will see, the spacetime of [7] has a conical singularity far from the string. Semiclassical gravity alone does not offer guidance towards its resolution. We will show that the CFT interpretation of the RS scenario suggests ways to resolve the singularity, which can have a semiclassical gravity description. We will investigate how the resolution of the singularity can affect low-energy quantities on the four dimensional world volume of the defect, notably the deviation of the gravitational potential from $\frac{1}{r}$.

To better elucidate the previous paragraph, we begin by noting that the 'local cosmic string' metric of [7] is that of AdS_6 with periodic identification of one of the coordinates:[1]

$$ds^2 = \frac{\omega^2}{R^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + R_0^2 d\theta^2\right) + R^2\frac{d\omega^2}{\omega^2}\ , \qquad (1)$$

where $\mu = 0, ..., 3$ are the Minkowski space coordinates, θ is an angular variable, and R is the radius of AdS_6. The "three brane" is placed at $\omega = R$ and space extends to the AdS horizon

[1]More precisely, this is the metric of a Poincaré patch of AdS. The coordinates we use are related to the ones of RS [1] as $\omega = Re^{-r/R}$.

$\omega = 0$ [7]. The proper size of the space transverse to the brane is infinite, but its volume is finite and the effective four-dimensional Planck mass is:

$$M_{Pl}^2 \sim M_6^4 \int_0^R d\omega \int_0^{2\pi} d\theta \sqrt{g} g^{00} \sim R_0 R M_6^4 \ , \tag{2}$$

where M_6 is the six-dimensional Planck mass. The authors of [7] showed that there is a graviton zero mode localized at $\omega = R$ and computed the correction to the gravitational potential in the four-dimensional theory due to the continuous spectrum of graviton "Kaluza-Klein" modes:

$$V_{(4)}(r) \sim \frac{m_1 m_2}{M_{Pl}^2} \frac{1}{r} \left(1 + \frac{R^3}{r^3} \right) \ , \tag{3}$$

where we omitted inessential numerical factors. The correction to Newton's law obtained in [7] has one extra power of r in the denominator compared to the corresponding correction in the five-dimensional RS scenario.

The calculation of (3) in [7] proceeded by imposing particular boundary conditions at the AdS horizon—the same as in the calculation in the codimension one RS case [1, 9]. There is an important difference between these two cases, however. The 'local cosmic string' spacetime (1) has a conical singularity at the horizon. To see this, note that the metric (1) is obtained from the AdS_6 metric upon identifying one of the Minkowski coordinates under the action of a discrete translation isometry of Minkowski space, $\theta \sim \theta + 2\pi$. Minkowski space translations, however, act non-freely on AdS, and it is easy to identify the fixed points with the Poincaré patch horizon (see [10] or Appendix A). Another way the singularity is seen is by noting that the proper radius of the circle parameterized by θ, $\mathcal{R}(\omega) = \omega R_0/R$, shrinks to zero at the horizon. The presence of the singularity (even though it is infinitely far away from the string) and its resolution can significantly affect the low-energy behavior of the theory on the defect (note that it takes a finite proper time for geodesics to reach the singularity).

In a string theory framework, as one approaches the horizon, closed-string winding modes become massless and the geometric description (1) becomes inadequate. The advantage of a string theory embedding is that, as we will see, it offers ways to deal with the singularity. In an effective field theory approach, on the other hand, there is a certain arbitrariness in the boundary conditions at the singularity, which feeds into the calculation of (3).

To get a guidance as to how the singularity might be resolved and what the low-energy consequences are, we note first that a dual interpretation can also be given to a RS setup with gravity localized on a five dimensional wall in AdS_6: five dimensional matter is coupled to a

"hidden" five dimensional CFT. This CFT gives rise to corrections to Newton's law identical to the ones computed from the classical cutoff-AdS_6 gravity. Wrapping one of the spacelike directions of the 5d boundary of AdS_6 on a circle, as in eqn. (1), corresponds to compactifying the dual 5d CFT (as well as the observable and 5d gravity sectors) on a circle. This breaks conformal invariance and induces a nontrivial renormalization flow of the 5d CFT to a 4d theory.[2] On general grounds, depending on the particular CFT and/or the details of the compactification, there appear to be three possibilities for the end point of this renormalization flow:

1. The 5d CFT on the circle flows to a 4d CFT in the IR. In this case, the resulting 4d effective theory is like that of the 4d Randall-Sundrum case—that of observable matter coupled via gravity to a "hidden" CFT. One expects the same power-law falloff of the corrections to Newton's law as in the codimension one case.

2. The 5d CFT flows to a confining 4d theory, which develops a mass gap, i.e. to a trivial infrared fixed point. In this case, one expects no power-law corrections to the gravitational potential in 4d. The resulting description is more like a conventional KK reduction, with discrete massive graviton modes.

3. The 5d CFT flows to a confining 4d theory, which dynamically breaks some global symmetries. In the infrared, there are weakly interacting massless degrees of freedom (goldstone, goldstino,...) in the hidden sector, giving rise to power-law corrections to the 4d gravitational potential in the visible sector similar to 1. above.

If a dual gravity description is applicable for any of these possible end points of the flow, 1. − 3. should correspond to modifications of the metric that resolve the singularity at the horizon. In what follows, we will show the existence of semiclassical gravity resolutions of the singularity dual to 1. and 2. above. It is not clear whether (in the deep infrared) 3. can have a semiclassical gravity description—the massless degrees of freedom in the field theory dual are weakly interacting at low energies and provide a weakly coupled description of the infrared physics; it is difficult to contemplate two different weakly coupled dual descriptions of the same physics.

[2]The flow of the 5d gravity to 4d is trivial—for R_0 greater than the 5d Planck length gravity is weakly coupled. In our discussion we will ignore the observable sector; we note only that obtaining chiral matter in 4d might require further orbifolding of the compactified direction.

This paper is organized as follows. In Sections 2 and 3, respectively, we consider resolutions of the singularity that correspond to the flows 1. and 2. above.

In Section 2, we follow the supergravity backgrounds corresponding to the various regions of the flow of the 5d CFT on a circle to a 4d CFT. We show that the singularity is replaced by a smooth horizon and the resulting metric, describing the deep infrared region, is nonsingular. This implies that the infrared physics is like that of the codimension one RS scenario. We consider the case of a flow of a 4d CFT on a circle to a 3d CFT in some detail, since the string theory embedding and relevant supergravity backgrounds are somewhat simpler, and then generalize to the 5d $\rightarrow$ 4d flow.

In Section 3, we consider a resolution of the singularity, which, in the dual field theory, corresponds to imposing antiperiodic boundary conditions on the fermions of the 5d CFT on the circle. The resulting theory flows to a 4d theory with a mass gap. We note that the use of string theory dualities in this particular resolution of the singularity, while suggestive, is not really needed (the resolution can be simply described in the framework of semiclassical gravity). We show that the resulting space is nowhere singular and that the long-distance physics on the defect is like that of conventional KK compactifications—there are only discrete massive graviton KK modes and no power-law corrections to Newton's law.

We conclude in Section 4. For completeness, we give various technical details, many of which can be found elsewhere in the literature [10, 11], in the appendices. In Appendix A, we show that the Minkowski translation isometries act non-freely on AdS, hence identifying the space under the action of a discrete translation leads to orbifold singularities [10]. In Appendix B, we consider the effect of the boundary conditions at the horizon on the low-energy physics on the defect, from an effective field theory point of view. Finally, in Appendix C, we derive the relation, used in Section 2, between the Neumann Green function, needed to compute the corrections to the gravitational potential in the RS scenario, and the Dirichlet kernel, used to compute correlators in the AdS/CFT correspondence.

2 Resolution of the singularity 1: flow to a 4d CFT

As suggested in [2] and further elaborated in [9, 11, 12, 13], a 4d dual description can be given to the RS scenario, in which the gravitational backreaction of a 3-brane in a 5d spacetime of constant negative curvature induces the localization of gravity near the brane. The effect of the noncompact bulk, *i.e.* of the continuum of "Kaluza-Klein" modes on the 3-brane physics,

can be reproduced in a purely 4d language by the presence of a (strongly coupled) “hidden” CFT, which couples to the 3-brane matter only gravitationally.

This picture is suggested by a generalization of the AdS/CFT correspondence. In the metric background

$$ds^2 = \frac{\omega^2}{R^2}\left(-dt^2 + \sum_{i=1}^{3} x^i x^i\right) + R^2\,\frac{d\omega^2}{\omega^2} \tag{4}$$

the 3-brane located at $\omega = R$ cuts off the region of AdS space with $\omega > R$. When one moves the 3-brane to infinity, the AdS/CFT correspondence relates the string theory partition function on the background (4) to the partition function of a 4-dimensional CFT “living” on the boundary of AdS [3, 4, 5]. Keeping the 3-brane at a finite ω is interpreted as imposing an UV cutoff on the CFT. An important consequence of having a finite cutoff is that the 4-dimensional theory includes gravity (which decouples when the cutoff is removed). Thus one can calculate the effects induced by loops of the cutoff CFT on the gravitational potential produced by a source. The authors of Ref. [12] have shown that these loops exactly[3] reproduce the results of Ref. [1]. In particular, the power-law falloff of the correction simply follows from the scaling of the two-point correlation function of the CFT stress-energy tensor at large distances (where the UV cutoff should not matter).

The above AdS/CFT interpretation of the Randall-Sundrum scenario suggests ways of resolving the conical singularity in (1). It is natural to interpret the corrections to Newton’s law (3) as arising from loops of a hidden 5d CFT with one dimension compactified on a circle. As was discussed in the Introduction, these corrections depend on the infrared behavior of this CFT.

In this Section, we will consider the first possible flow of the 5d CFT on a circle described in the Introduction—that to a 4d CFT. We begin by studying first a simpler problem—the flow of a 4d CFT on a circle to a 3d CFT. The dual gravity description is that of a wrapped 3-brane in a 5-dimensional universe (times a compact manifold).[4] All the issues regarding the singularity are the same as in the case of a wrapped 4-brane; as we will see, the resolution of the singularity is also very similar.

We begin by considering the type IIB supergravity solution, corresponding to a stack of N

[3]Due to a nonrenormalization theorem, see [14], the one loop result for the two-point function of the stress energy tensor of the $N = 4$ SYM theory is exact and applies in the strong coupling limit, where the comparison to the semiclassical gravity calculation is appropriate.

[4]Here the corrections to Newton’s law cannot really be considered small since at low energies the wrapped 3-brane looks 2-dimensional and the leading term is logarithmic. Nevertheless, it is useful to study first this case since the supergravity duals describing the flow of the CFT to the infrared can be described rather explicitly.

coincident D3-branes wrapped on a circle of radius R_0:

$$ds^2 = H(r)^{-1/2}\left(-dt^2 + \sum_{i=1}^{2} x^i x^i + R_0^2\, d\theta^2\right) + H(r)^{1/2}\left(dr^2 + r^2 d\Omega_5^2\right) \tag{5}$$

where $H(r) = 1 + \frac{R_3^4}{r^4}$ and $R_3 = (4\pi g_{YM}^2 N)^{\frac{1}{4}} l_s$ is assumed to be large enough so that the supergravity approximation can be trusted.

In the near-horizon limit ($l_s \to 0$, r/l_s^2-fixed) we can neglect the 1 in $H(r)$; the metric Eq. (5) then reduces to

$$ds^2 = l_s^2\left[\frac{u^2}{\hat{R}_3^2}\left(-dt^2 + \sum_{i=1}^{2} x^i x^i + R_0^2\, d\theta^2\right) + \hat{R}_3^2\,\frac{du^2}{u^2} + \hat{R}_3^2\, d\Omega_5^2\right]. \tag{6}$$

where $\hat{R}_3$ is the AdS radius in units of the string length l_s and following [3] we expressed the metric in terms of the "energy" variable $u = \frac{r}{l_s^2}$.

We note that Eq. (6), with the S^5 integrated out, is the same as (1) (less one Minkowskian dimension), if we appropriately restrict the range of u. As in Ref. [7] we can think of this metric as describing the solution outside of a wrapped 3-brane—the "Planck" brane, not to be confused with the stack of N wrapped D3 branes whose near-horizon limit is Eq. (6)—located at $u_0 = \hat{R}_3 l_s^{-1}$. The metric Eq. (6) localizes gravity close to u_0 in the same manner as in the original RS scenario. We are interested in the gravitational potential due to a source on the brane when probed by matter living also on the brane, and more specifically in the corrections to the (in this case, 2-dimensional) Newtonian potential induced by the presence of the noncompact bulk. In the spirit of the AdS/CFT correspondence, we can think of this scenario as being dual to a 4d theory where the effects of the bulk are replaced by a cutoff CFT, weakly coupled to gravity [2, 9, 11, 13]. Note, however, that the conformal invariance is broken not only by the cutoff but also by the fact that one of the dimensions is compactified on a circle.[5] We want to show that the 4d theory flows to a nontrivial infrared fixed point, where the corrections to Newton's law can be easily estimated.

In order to do this recall that the UV/IR correspondence [15, 16] relates the low-energy physics in the CFT to the physics of the "small-u" region of the supergravity theory in the background Eq. (6). More precisely, the Newtonian potential at (X, u) due to a pointlike source at $(0, u)$ can be obtained from the Green function $G_N(X, 0; u)$ with Neumann boundary

[5]The transformation $(u, x^i, \theta) \to (\lambda^{-1}u, \lambda x^i, \lambda\theta)$ fails to be an isometry of the metric Eq. (6) because it changes the range of θ from $(0, 2\pi)$ to $(0, 2\pi\lambda)$.

conditions imposed at u [9]. Here X is shorthand for (x^i, θ). But as shown in Ref. [17], if $k \ll \hat{R}_3^{-2} u$ we have[6]

$$\tilde{G}_N(k; u_0) = Z^2(u)\, \tilde{G}_N(k; u) \left(1 + O\left(\frac{\hat{R}_3^2 k}{u}\right)\right) \tag{7}$$

where $k = \left(k^i, \frac{n}{R_0}\right)$ is the momentum conjugate to X. In other words, up to a wavefunction renormalization $Z^2(u)$, the leading k dependence in $\tilde{G}_N(k; u_0)$ (whose spatial Fourier transform yields the gravitational potential of a static source at u_0) comes from the region $u \gtrsim \hat{R}_3^2 k$.

On the other hand, from the metric Eq. (6), we see that at any given u the proper radius of the compact dimension is $\mathcal{R}(u) = \frac{(l_s u)}{\hat{R}_3} R_0$ and therefore the mass of the KK modes is $m_{KK}(u) \sim \frac{1}{\mathcal{R}(u)}$, while the mass of the winding modes is $m_w = \frac{\mathcal{R}(u)}{l_s^2}$. These two masses become comparable at $u_* \equiv \frac{\hat{R}_3}{R_0}$. It follows that when $k \lesssim \frac{u_*}{\hat{R}_3^2} = \frac{1}{\hat{R}_3 R_0}$, the supergravity approximation must break down.

In fact, for $u R_0 \ll \hat{R}_3$, it is more appropriate to use the T-dual description in terms of a D2-brane localized on a circle of radius $\tilde{R}_0 = \frac{l_s^2}{R_0}$. The following analysis is very similar to the one presented in Ref. [18] except that now we have one compact dimension. For the sake of completeness, we summarize the main steps. The type IIA supergravity background corresponding to the T-dual stack of D2-branes localized on a circle of radius $\tilde{R}_0$ is:

$$ds^2 = H(\bar{r})^{-1/2} \left(-dt^2 + \sum_{i=1}^{2} x^i x^i\right) + H(\bar{r})^{1/2} \left(d\bar{r}^2 + \bar{r}^2 d\Omega_6^2\right) \ , \tag{8}$$

and the dilaton field is given now by:

$$e^{\Phi(\bar{r})} = g_s H(\bar{r})^{\frac{1}{4}}. \tag{9}$$

To impose the correct periodicity we take, in the near-horizon limit:

$$H(\bar{r}) = \sum_{n=-\infty}^{\infty} \frac{R_2^5}{\mid \bar{r} - \bar{r}_n \mid^5} \tag{10}$$

where $\bar{r}_n \equiv (x_3, x_4, \ldots, x_9) = (0, 0, \ldots, 2\pi n \tilde{R}_0)$ and $R_2 = (6\pi^2 g_s N)^{\frac{1}{5}} l_s$. We also identify $r^2 = \bar{r}^2 - x_9^2$ with the coordinate appearing in Eq. (5). Defining as before energy variables by $u = \frac{r}{l_s^2}$, $\bar{u} = \frac{\bar{r}}{l_s^2}$ and $u_9 = \frac{x_9}{l_s^2}$, we find after Poisson resummation that:

$$H(r) = \frac{\hat{R}_3^4}{(l_s u)^4} \left\{1 + 2\sum_{m=1}^{\infty} (m u R_0)^2 \cos(m u_9 R_0) K_2(m u R_0)\right\} \tag{11}$$

[6] The authors of Ref. [17] were interested in the two-point correlation function $A(k^2) = \int dx e^{ikx} \langle \mathcal{O}(x) \mathcal{O}(0) \rangle$. This is related to the Neumann Green function by $G_N(k; u_0) = -A(k^2)^{-1}$ (see Appendix C). They also expressed their results in terms of the coordinate $z = \frac{\hat{R}_3^2}{u}$.

where $K_2(x)$ is a modified Bessel function of the second kind and we used $g_{YM}^2 = \frac{R_0}{l_s} g_s$ (with g_s—the type IIA string coupling). This expression shows that in the limit $uR_0 \gg 1$ we have $H \simeq \frac{\hat{R}_3^4}{(l_s u)^4}$ up to exponential corrections, and the metric Eq. (8) can be written as:

$$ds^2 = l_s^2 \left[\frac{u^2}{\hat{R}_3^2} \left(-dt^2 + \sum_{i=1}^{2} x^i x^i \right) + \frac{\hat{R}_3^2}{u^2} \left(du^2 + u^2 d\Omega_5^2 + \frac{1}{R_0^2} \, d\theta^2 \right) \right]. \tag{12}$$

We see that in the T-dual description, the proper radius of the compact dimension parameterized by $x_9 = \tilde{R}_0 \, \theta$ shrinks at large u instead—it becomes of order l_s when $uR_0 \sim \hat{R}_3$. This agrees with the limit of validity $uR_0 > \hat{R}_3$ found in the wrapped D3-brane background Eq. (6).

The D2-brane gravity background breaks down, in its turn, at small u—this reflects the nonconformality of the D2 brane world volume theory, which becomes strong in the infrared. To see this, note that in the opposite limit $uR_0 \ll 1$, we can just keep the $n = 0$ term in Eq. (10). One finds that $H \simeq \frac{\hat{R}_2^5}{(l_s \bar{u})^5}$ and the effective string coupling e^Φ becomes of order one at $uR_0 \sim \frac{(6\pi^2)^{1/5}}{N^{4/5}} (g_{YM}^2 N)$. In this energy regime, the supergravity dual is eleven dimensional—we can uplift the 10-dimensional D2-brane background to a solution of eleven dimensional supergravity; for details see [18]. This solution is, in its turn, the limit of the M2-brane background when the distances involved are much larger than $R_{11} = g_s l_s$. Since we are interested in the deep infrared, we will skip the uplifted D2 brane dual and go directly to the M2-brane description.

The near-horizon metric of a stack of N M2-branes is given by:

$$ds^2 = f(\tilde{r})^{-2/3} \left(-dt^2 + \sum_{i=1}^{2} x^i x^i \right) + f(\tilde{r})^{1/3} \left(d\tilde{r}^2 + \tilde{r}^2 d\Omega_7^2 \right) \ , \tag{13}$$

$$f(\tilde{r}) = \sum_{n,m=-\infty}^{\infty} \frac{R^6}{\mid \tilde{r} - \tilde{r}_{n,m} \mid^6} \ , \tag{14}$$

where $\tilde{r}_{n,m} \equiv (x_3, \ldots, x_9, x_{10}) = (0, \ldots, 2\pi n \tilde{R}_0, 2\pi m R_{11})$ and $R = (32\pi^2 N)^{\frac{1}{6}} l_P$ (with l_P the 11d Planck length). Now we identify $\bar{r}^2 = \tilde{r}^2 - x_{10}^2$ with the coordinate in Eq. (8). This metric has various limits depending on the relative size of R_{11} and $\tilde{R}_0$. However, we are interested in the deep infrared dynamics of the CFT, which is mapped to the region $r \ll R_{11}, \tilde{R}_0$. In this regime, the harmonic function Eq. (14) becomes $f(\tilde{r}) \simeq \frac{R^6}{\tilde{r}^6}$ and the metric Eq. (13) can be written as

$$ds^2 = \frac{\omega^2}{R^2} \left(-dt^2 + \sum_{i=1}^{2} x^i x^i \right) + R^2 \, \frac{d\omega^2}{\omega^2} + R^2 \, d\Omega_7^2 \ , \tag{15}$$

where we defined the new variable $\omega = \frac{\tilde{r}^2}{R}$. This metric describes just $AdS_4 \times S_7$ which shows that there are no further singularities. This supergravity background is conjectured to be dual to a 3d CFT.

To summarize, the previous chain of arguments describes—via the AdS/CFT correspondence—the flow of a 4d theory which is approximately conformal in the ultraviolet (where one can ignore the fact that one dimension is compactified on a circle), to a 3d CFT in the infrared. If one interprets the corrections to Newton's law as arising from loops of this theory, it is appropriate to calculate the effects of the infrared CFT. The leading correction comes from the two-point correlation function of the CFT stress-energy tensor. But for a 3-dimensional conformal theory

$$\langle T(x)T(0)\rangle \sim \frac{c}{x^6} \ , \tag{16}$$

where $c \sim (R/l_P)^9 \sim N^{\frac{3}{2}}$ is the central charge [14], which induces a correction to the Newtonian potential of order

$$\Delta V(r) = \frac{1}{M_3}\int d^2\!p\, e^{-ipx}\frac{1}{p^2}\langle T(p)T(-p)\rangle\frac{1}{p^2} \sim \frac{c}{M_3}\int d^2\!p\,\frac{e^{-ipx}}{p} \sim \frac{c}{M_3}\frac{1}{r} \ , \tag{17}$$

where M_3 is the Planck scale of the 3d gravity theory. We see that the stringy resolution of the conical singularity has a dramatic effect on the power-law corrections to the gravitational potential since a naïve effective field theory approach would suggest corrections of order r^{-2}, similar to (3).

Let us now describe the corresponding construction for the wrapped 4-brane case. The considerations here closely parallel the ones for the wrapped 3-brane and we will be correspondingly brief.

The string background whose near-horizon limit is a 6d AdS space (times a compact space) is that of the type I' D4-D8 brane system [20]. This background is dual—in the large-N limit, where N is the number of D4 branes—to a strongly coupled five dimensional supersymmetric CFT with an E_{N_f+1} global symmetry ($N_f < 8$ is the number of D8 branes at the O8 plane; for details, see [19, 20]). The gravity background is a fibration of AdS_6 over S^4 and is a solution of massive type IIA supergravity [20, 21].

The $AdS_6 \times S^4$ gravity background, therefore, provides a starting point to study the string embedding of (1). The metric (1) thus represents the (wrapped) AdS part of the D4-D8 near-horizon geometry, with one world-volume direction wrapped on a circle. The restriction of the radial AdS coordinate to $\omega < R$ is again, in the spirit of the UV/IR correspondence, interpreted as imposing a cutoff on the dual 5d CFT. The 5d Newtonian potential, corrected by loops from this "hidden" CFT, can be easily evaluated and (ommitting numerical factors) yields, in the unwrapped ($R_0 \to \infty$) limit:

$$V_{(5)}(r) \sim \frac{m_1 m_2}{M_5^3}\frac{1}{r^2}\left(1+\frac{R^3}{r^3}\right) \ , \tag{18}$$

where $M_5^3 \sim M_6^4 R$ is the 5d Planck scale. The power-law falloff of the correction with r can be obtained by scaling from the two-point function of the energy momentum tensor of the 5d CFT, as in (17) above. We conclude that the scaling of the correction in eqn. (3) is appropriate at distances $r \ll R_0$, where the breaking of conformal invariance is inessential.

As in our discussion of the 4d $\to$ 3d flow above, wrapping one world volume direction on the circle breaks conformal invariance and induces a nontrivial renormalization group flow to a 4d theory. As in going from Eq. (6), via Eqs. (8) and (13), to Eq. (15) above, we can use T-duality to study the dual gravity description of the flow of the compactified CFT to the infrared. T-duality along the wrapped worldvolume direction maps the D4-D8 brane system to the D3-D7 system on a transverse circle of radius $\tilde{R}_0 = l_s^2/R_0$. At energies below $1/R_0$ in the CFT (corresponding to radial distances $\ll \tilde{R}_0$) the $\tilde{R}_0$ circle is irrelevant and the geometry is approximately that of the near horizon limit of the D3-D7 gravity background, which has been studied in [22, 23]. The deep infrared metric background is $AdS_5 \times \tilde{S}^5$ (the space $\tilde{S}^5$ can be described as an S^5, but with unusual periodicity of one of the angular variables; for details, see [22, 23]). This shows that the singularity is resolved and is replaced by a smooth horizon as in Eq. (15), as appropriate in the dual description of a theory flowing to an IR fixed point. Since, by the UV/IR correspondence, the nonsingular near-horizon region is the one relevant for the deep infrared physics, we expect that the infrared physics on the wrapped 4-brane of [7]—with this particular resolution of the singularity—is like that of the codimension one RS scenario.

3 Resolution of the singularity 2: flow to a 4d theory with mass gap

Another way to resolve the singularity is to modify the metric in the interior, away from the "brane" at $\omega = R$, in such a way that the conical singularity of (1) is hidden behind a smooth horizon.[7] In the dual CFT this modification of the metric has the interpretation of imposing supersymmetry breaking (antiperiodic on the fermions) boundary conditions on the compactified circle [24]. The dual 5d CFT then flows to a nonsupersymmetric pure Yang-Mills theory in 4d. The latter has a mass gap and one does not expect long-range (power-law) deviations from Newton's law. Thus, one expects that this type of localization of gravity is rather similar to the usual KK reduction—there are only discrete KK modes and no power-law corrections to the gravitational potential. This resolution of the singularity realizes the

[7]This line of thought was suggested to us by S. Trivedi.

possibility 2. pointed out in the Introduction. In this Section, we consider the resolution of the singularity by smoothing out the metric in the interior and show how the above expectations for the infrared physics are borne out on the gravity side.

We begin by considering the most general solution of the vacuum Einstein equations in AdS_6 with an $SO(2) \times ISO(1,3)$ isometry [25, 26, 27]:

$$ds^2 = \frac{\omega^2}{R^2}\,\eta_{\mu\nu}dx^\mu dx^\nu + R^2 \frac{d\omega^2}{\omega^2\left(1-\frac{b^5}{\omega^5}\right)} + \frac{\omega^2}{R^2}\left(1-\frac{b^5}{\omega^5}\right)R_0^2 d\theta^2 \ , \tag{19}$$

where b is a yet to be determined constant of integration and θ is periodic with period 2π. For $b = 0$ this is just the metric (1). It is easy to see that the metric (19) admits half the Killing spinors of AdS_6 for $b = 0$ [28] and no Killing spinors at all if $b \neq 0$. The analytic continuation ($\theta \to it/R_0$, $x^0 \to ix^4$) of the metric (19) can be obtained by a scaling limit from the AdS_6 Schwarzschild black hole solution [29, 21]; the dual CFT interpretation of this metric background mentioned above is a direct consequence of this [24].

If $b \ll R$, for $b \ll \omega \leq R$ the metric approximates that of [7]. On the other hand, at $\omega \sim b$ the spacetime is significantly changed; in particular, for general values of b, the metric has a conical singularity at $\omega = b$. This singularity can be avoided if one makes a particular choice of b. By considering (19) near $\omega = b$, it is easily seen that the deficit angle singularity is absent for

$$b = \frac{2}{5}\,\frac{R^2}{R_0} \ , \tag{20}$$

and the metric of the 2d transverse space parameterized by (ω, θ) becomes near b that of the plane in polar coordinates. The point $\omega = b$ is then a nonsingular horizon. The singularity of (19) at $\omega = 0$ is thus hidden "behind" the horizon; we should note that this language may be a bit misleading: for b given by (20) and $b \leq \omega \leq R$ the space (19) is complete and nonsingular—there is no region "behind" the horizon $\omega = b$. From now on we will consider the particular value (20) of b and discuss the implications for the "localization" of gravity and the corrections to Newton's law on the $\omega = R$ brane.[8]

In fact, most of the relevant analysis already exists in the literature: an analogous construction, using compactification of the $M5$ brane theory on $S^1 \times S^1$ with supersymmetry breaking boundary conditions on one of the S^1, was used to study glueball masses in QCD_4 via a scaling

[8] We note that for b given by (20), the relation between the 4d and 6d Planck scales is: $M_4^2 \sim M_6^4 \int \sqrt{g} g^{00} \sim M_6^4 R R_0 \left[1 - \left(\frac{2R}{5R_0}\right)^3\right] \simeq M_6^4 R R_0$ and it reproduces (2) for $b \ll R$.

limit of the AdS_7 black hole [24, 30]. The only modification here is that by considering only $\omega < R$ we cutoff the boundary region of AdS; correspondingly, as in the RS scenario, we need to consider the non-normalizable modes as well.

To this end, consider the massless scalar wave equation for $\phi(k,n;\omega)$ in the background (19):

$$\frac{1}{R^2\,\omega^2}\,\partial_\omega\left[\omega^6\left(1-\frac{b^5}{\omega^5}\right)\partial_\omega\phi(k,n;\omega)\right]-\frac{\omega^5}{\omega^5-b^5}\frac{n^2R^2}{R_0^2}\,\phi(k,n;\omega)-k^2R^2\,\phi(k,n;\omega)=0\ , \quad (21)$$

where we have Fourier transformed the field ϕ with respect to the 4d coordinates and θ. The boundary condition at $\omega = b$ plays a crucial role in determining the allowed k^2 for the solutions of this equation. To determine the boundary condition, note that near $\omega = b$ the Laplacian in (21) becomes the Laplace operator, ∇^2, on the plane (with radial coordinate $\rho^2 = \frac{4}{5}R^2(\frac{\omega}{b}-1)$) and the operator acting on ϕ is $\sim \nabla^2 - \kappa^2$, with $\kappa^2 \sim k^2$. The solutions of this equation are $J_0(\kappa\rho)$ and $N_0(\kappa\rho)$. The N_0 solution must be discarded, as it yields a delta-function singularity ($N_0(x) \sim \log x$ near $x = 0$) when acted upon with ∇^2 and hence does not obey (21) near $\omega = b$. Keeping only the J_0 solution is equivalent to imposing $\partial_\omega\phi(k,n;\omega = b) = 0$.

The values of k^2 for which the solutions of (21) obey the Neumann boundary condition at $\omega = b$ determine the k^2-plane poles of the boundary-to-boundary Neumann Green function $\tilde{G}_N(k;\omega,\omega')|_{\omega=\omega'=R}$. These poles, in turn, determine the masses of the 4d excitations propagating on the defect. For $k^2 = 0, n = 0$, the solution is clearly $\phi = const$. This is nonnormalizable in the infinite AdS case, but is normalizable in the $\omega < R$ slice. The contribution of the constant solution gives rise to the massless graviton and to Newton's law in the four dimensional theory on the $\omega = R$ boundary. Since, as in [24, 30], there is no continuum of allowed values of k^2 (the quantization occurs because of the Neumann boundary condition at $\omega = b$) we do not expect the nonanalytic behavior—a logarithmic cut starting at $k^2 = 0$ in $\tilde{G}_N(k;R,R)$—that leads to power-law correction in the RS case. One expects then that the leading correction is exponential, due to exchange of heavy KK states.

4 Concluding remarks

We considered the localization of gravity in codimension two, on a "stringlike defect" in six dimensions [7]. We showed, guided by the AdS/CFT correspondence, that the resolution of the singularity of the metric at the horizon can affect the low-energy physics on the defect and change drastically the long-distance corrections to Newton's law. This has a natural interpre-

tation in the dual CFT description—the long distance corrections to the gravitational potential due to hidden CFT loops clearly depend on the infrared physics of the CFT, since compactifying on the circle breaks conformal invariance and induces a nontrivial flow. We enumerated various possibilities and considered two examples of resolutions of the singularity that have a semiclassical gravity description.

Singularities like the one considered in this paper will occur also in generalizations to higher codimension [8]. In particular, one can wrap $d-4$ of the spatial Minkowski coordinates of the Poincaré patch of AdS_{d+1} on a Ricci flat compact manifold. The corresponding generalization of (1) is still a solution of the vacuum AdS_{d+1} Einstein equations and leads to localization of gravity. As one approaches the horizon, the size of the compact manifold shrinks, invalidating thus the gravity description. In a dual CFT language, similar to the case considered here, a nontrivial renormalization flow of the CFT is induced. One expects that in each case the nature of the infrared dynamics—depending on the details of the compactification—will influence the long-distance physics on the defect.

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6 Appendix A:

Here we show that the group of spatial Minkowski translations acts non-freely on AdS. Therefore, identifying AdS points related by the action of these isometries leads to singularities at the fixed points.

To see this, recall that AdS_{p+2} of unit radius is defined as the hyperboloid:

$$X_0^2 + X_{p+2}^2 - \sum_{i=1}^{p+1} X_i^2 = 1 \ , \tag{22}$$

embedded in $p+3$ dimensional flat space with signature $(-,+,...,+,-)$. The Poincaré coordinates $\omega, t, \vec{x}$, with $\vec{x} = (x_1, ..., x_p)$ and $0 < \omega < \infty$, cover half the hyperboloid:

$$X_0 - X_{p+1} = \omega$$

$$
\begin{aligned}
X_0 + X_{p+1} &= \frac{1}{\omega} + \vec{x}^2\,\omega - t^2\,\omega \\
X_i &= x_i\,\omega\ , \quad i = 1, ..., p, \\
X_{p+2} &= t\,\omega \quad .
\end{aligned}
\tag{23}
$$

From (23) it is easy to see that a Minkowski spatial translation $\vec{x} \to \vec{x}+\vec{a}$ acts on the hyperboloid as:

$$
\begin{aligned}
X_0 - X_{p+1} &\to X_0 - X_{p+1} \\
X_0 + X_{p+1} &\to X_0 + X_{p+1} + \vec{a}^2(X_0 - X_{p+1}) + 2\vec{a}\cdot\vec{X} \\
X_i &\to X_i + a_i(X_0 - X_{p+1})\ ,\ i = 1, ..., p, \\
X_{p+2} &\to X_{p+2} \quad .
\end{aligned}
\tag{24}
$$

Therefore, translations leave the following points on the hyperboloid invariant:

$$
X_{p+1} = X_0\ , \quad \vec{a}\cdot\vec{X} = 0\ . \tag{25}
$$

Using the map (23), the fixed points are easily identified with the Poincaré horizon $\omega = 0$. Thus identifying AdS_{p+2} under the action of a discrete translation leads to conical singularities at the horizon.

7 Appendix B:

From an effective 6d gravity point of view the result that the long distance correction to Newton's law is determined primarily by the near-horizon geometry, *i.e.* by the resolution of the singularity, is somewhat puzzling. The point is that the calculation of the potential in the wrapped AdS_6 geometry of ref. [7] appears to be insensitive to the singularity—recall that the singularity is of the orbifold type, i.e. no curvature invariants blow up as one approaches the horizon. In this Appendix, we study this issue and point out that the leading correction to the gravitational potential on the defect depends on the boundary conditions imposed "at the singularity." From a low-energy point of view, there appears to be an arbitrariness in the choice of boundary conditions, reflecting the ignorance of the naïve low-energy theory on the mechanism resolving the singularity.

The calculation of the correction to Newton's law can be performed by finding the graviton boundary-to-boundary Green function in AdS_{d+1} obeying certain boundary conditions; this is

equivalent to the calculations of [1] (and of [7] for the wrapped case) done by decomposing into graviton "KK" modes. Since the appropriate graviton propagator can be expressed in terms of the massless scalar propagator [9], for our purpose it will be sufficient to study the scalar Green function.

We begin by studying the massless scalar Green function in the wrapped AdS_{d+1} background with particular attention to the boundary conditions. We use the metric[9]

$$ds^2 = \frac{1}{z^2}\eta_{\mu\nu}dx^\mu dx^\nu + R^2\frac{dz^2}{z^2} \ , \tag{26}$$

where $\eta_{\mu\nu}$ is the d dimensional Minkowski metric (mostly plus). To study the wrapped case, we will imagine that one of the $d-1$ spatial coordinates is compactified on a circle of coordinate radius R_0, as in (1). We will let the coordinate z change between $z = \epsilon$ and $z = \infty$ (the AdS horizon in these coordinates); thus the "brane" is located at $z = \epsilon$. We denote the Fourier transform of the massless scalar field with respect to the Minkowski coordinates by $\phi(k, z)$, where k is the d-dimensional momentum vector (with a discrete component n/R_0 when a direction is compactified). The scalar Laplace operator ($\nabla^2 \equiv g^{-1/2}\partial_M g^{MN} g^{1/2}\partial_N$; $g^{1/2} \equiv R z^{-d-1}$) thus becomes:

$$\nabla_z^2\phi(k,z) = \frac{z^2}{R^2}\phi''(k,z) - \frac{(d-1)z}{R^2}\phi'(k,z) - k^2z^2\phi(k,z) \ , \tag{27}$$

where primes denote derivatives with respect to the z coordinate. For any two fields $\phi_1(k, z)$ and $\phi_2(k, z)$ we can, upon using (27) and integrating by parts, obtain the Green formula:

$$\begin{aligned}&\int_\epsilon^\infty dz\sqrt{g}\left[\phi_2(k,z)\nabla_z^2\phi_1(k,z) - \phi_2(k,z)\nabla_z^2\phi_1(k,z)\right] \\ &= \sqrt{g}\, g^{zz}\left[\phi_2(k,z)\partial_z\phi_1(k,z) - \phi_2(k,z)\partial_z\phi_1(k,z)\right]\Big|_{z=\epsilon} \ .\end{aligned} \tag{28}$$

If we take now $\phi_1(k, z) = G(k; z, z')$, obeying

$$\sqrt{g}\,\nabla_z^2 G(k;z,z') = \delta(z-z') \ , \tag{29}$$

and $\phi_2(k, z) = \phi(k, z)$—a solution of the bulk equation $\nabla_z^2\phi(k, z) = J(k, z)$, and substitute in (28) we obtain:

$$\begin{aligned}\phi(k,z') &= \int_\epsilon^\infty dz\sqrt{g}\,G(k;z,z')J(k,z) \\ &+ \sqrt{g(z)}\,g^{zz}(z)\left[\phi(k,z)\partial_z G(k;z,z') - G(k;z,z')\partial_z\phi(k,z)\right]\Big|_{z=\epsilon} \ .\end{aligned} \tag{30}$$

[9]This metric is related to the metric in (1) by the change of variables $z = \frac{R}{\omega}$. This parametrization is more convenient for the purpose of deriving the propagator.

The Green formula (30) is important in determining the consistency of various boundary conditions on ϕ and G at $z=\epsilon$.

The calculation of the "Newtonian" potential—here we really are computing the scalar Green function; for its relation to the graviton one, see [9]—requires finding the Green function obeying a Neumann (N) boundary condition $\partial_z G_N(k;z,z')|_{z=\epsilon}=0$ at $z=\epsilon$. One then computes the boundary-to-boundary Green function $G_N(k;\epsilon,\epsilon)$ for spacelike momenta (with $k_0=0$). The potential at the boundary, $\varphi(r,\epsilon)$, due to a static source J at $z=\epsilon$ is found from eqn. (30)[10] after a $d-1$ dimensional spatial Fourier transform. If a direction is wrapped, the Fourier transform w.r.t. the wrapped component of k is replaced by a discrete sum. In order to find the leading long-distance behavior of $V(r)$, we need the small-k expansion of the boundary-to-boundary propagator $G_N(k;\epsilon,\epsilon)$. The k^{-2} term yields the leading term, giving Newton's law in d dimensions after the Fourier transform. The terms containing positive integer powers of k^2 yield local terms in $V(r)$—delta function and its derivatives—and thus do not affect the long-distance behavior. The leading long-distance correction to Newton's law arises from the first nonanalytic term—logarithm of k^2 for d-even or a fractional power of k^2 for odd d.

Even though the horizon at $z=\infty$ is not a boundary in the same sense as the hyperplane $z=\epsilon$, the Poincaré patch parameterized by Eq.(26) can be continued beyond the horizon and it is necessary to specify appropriate boundary conditions in order to obtain a unique Green function. We are interested in exploring the possible low-energy effects parameterized by this freedom.

To begin, note that the general solutions of (29) for $z\neq z'$ are:

$$\begin{aligned} G_<(k;z,z') &= f_1(k;z')z^{\frac{d}{2}}H^{(1)}_{\frac{d}{2}}(ikRz)+f_2(k;z')z^{\frac{d}{2}}H^{(2)}_{\frac{d}{2}}(ikRz)\ ,\ \text{for } z<z'\ , \\ G_>(k;z,z') &= g_1(k;z')z^{\frac{d}{2}}H^{(1)}_{\frac{d}{2}}(ikRz)+g_2(k;z')z^{\frac{d}{2}}H^{(2)}_{\frac{d}{2}}(ikRz)\ ,\ \text{for } z>z'\ , \end{aligned} \tag{31}$$

where $k=\sqrt{k^2}$ for spacelike Minkowski momenta and $k=i\sqrt{|k^2|}$ for timelike momenta. We note that since $H^{(2)}(y)\sim e^{-iy}$ as $y\to\infty$, the second term in $G_>$ exponentially grows for spacelike momenta as z approaches the horizon. For timelike momenta and positive frequency k_0 the $H^{(2)}$ term in $G_>$ represents a wave moving in from the past horizon ($\sim e^{ik_0t+i|k|Rz}$), while the first term represents a wave traveling towards the future horizon.

[10] The Neumann boundary condition at $z=\epsilon$ assures that the first boundary term in (30) does not contribute. One is then free to specify the normal derivative, $\partial_z\phi(k,z)$, at the boundary. In the cases of interest one imposes a Z_2 reflection symmetry about the brane at $z=\epsilon$, so the normal derivative of $\phi(k,z)$ at the boundary actually vanishes and the potential is just given by the bulk integral in (30).

In the calculation of ref. [9] the Hartle-Hawking boundary condition, corresponding to keeping only the wave moving towards the future horizon—in other words setting $g_2 \equiv 0$ in (31)—was imposed. This is also the natural boundary condition to impose in D-brane absorption cross section calculations that lead to the AdS/CFT correspondence [4].

In what follows, we will proceed without imposing the Hartle-Hawking boundary condition at the horizon. The rationale is that the resolution of the singularity at the horizon changes the "potential" in the near horizon region and induces a "reflected" wave; therefore a more general condition, allowing for both reflected and transmitted waves, should be imposed at large z.

Imposing now the Neumann condition $\partial_z G_N(k;z,z')|_{z=\epsilon} = 0$ on $G_<$ and the appropriate discontinuity at $z = z'$ to reproduce the delta function in (29), we find that the Neumann function G_N is given by (31), where g_1 and $f_{1,2}$ are:

$$\begin{aligned}
g_1(k;z) &= \frac{1}{W(k;\epsilon)}\left(M_2(k;z) - M_1(k;z)\frac{M_{2,z}(k;\epsilon)}{M_{1,z}(k;\epsilon)}\right) - g_2(k;z)\frac{M_{2,z}(k;\epsilon)}{M_{1,z}(k;\epsilon)} , \\
f_1(k;z) &= g_1(k;z) - \frac{M_2(k;z)}{W(k;\epsilon)} , \\
f_2(k;z) &= g_2(k;z) + \frac{M_1(k;z)}{W(k;\epsilon)} ,
\end{aligned} \tag{32}$$

where we defined the functions $M_{1,2}(k;z)$ as

$$M_{1[2]}(k;z) \equiv z^{\frac{d}{2}} H^{(1)[(2)]}_{\frac{d}{2}}(ikRz) , \tag{33}$$

and $M_{1[2],z}(k;z) \equiv \partial M_{1[2]}(k;z)/\partial z$. The function $g_2(k;z)$ is arbitrary for now; as $g_2 \to 0$ we obtain the Hartle-Hawking Green function given in [9]. In eqn. (32) we have introduced

$$W(k;z) \equiv \frac{M_2(k;z)M_{1,z}(k;z) - M_1(k;z)M_{2,z}(k;z)}{Rz^{d-1}} , \tag{34}$$

which, being the Wronskian of two solutions of the homogeneous equation, is constant (this can be inferred from (28)) and can be evaluated, for example at the boundary at $z = \epsilon$. From eqn. (30), it follows that for consistency the Green function has to also satisfy eqn. (29) when the Laplacian acts on the second argument. This implies, upon inspection of (32), that the function $g_2(k;z)$ has to also solve the homogeneous Laplace equation, *i.e.*

$$g_2(k;z) = a(k)M_1(k;z) + b(k)M_2(k;z) , \tag{35}$$

where $M_{1,2}$ were defined in (33) and $a(k), b(k)$ are still arbitrary functions of momentum (we note that for any $g_2(k;z)$ of the above form one obtains the correct discontinuity of $\partial_{z'}G(k;z,z')$

at $z = z'$). Consistency of the Green formula (30) requires also that $\partial_{z'} G_>(z, z')|_{z'=\epsilon} = 0$ leading to the relation

$$a(k) = -b(k)\,\frac{M_{2,z}(k;\epsilon)}{M_{1,z}(k;\epsilon)}\,, \tag{36}$$

leaving one arbitrary function of momentum, $b(k)$, in the Green function. Putting Eqs.(31), (32), (35), and (36) together we get the Neumann Green function

$$\begin{aligned} G_N(k;z,z') \;&=\; \frac{M_1(k;z_>)}{W(\epsilon)}\left(M_2(k;z_<) - M_1(k;z_<)\frac{M_{2,z}(k;\epsilon)}{M_{1,z}(k;\epsilon)}\right) \\ &\quad + b(k)\left(M_2(k;z) - M_1(k;z)\frac{M_{2,z}(k;\epsilon)}{M_{1,z}(k;\epsilon)}\right)\left(M_2(k;z') - M_1(k;z')\frac{M_{2,z}(k;\epsilon)}{M_{1,z}(k;\epsilon)}\right) \end{aligned} \tag{37}$$

where $z_>$ ($z_<$) is the greatest (smallest) of z, z'. This formula displays the fact that the general solution can be written as the sum of the Green function obeying the Hartle-Hawking boundary condition (given by the first term) plus an arbitrary solution of the homogeneous equation, given by the second term.

In this language the resolution of the singularity would amount to the specification of the boundary conditions at the horizon which would determine $b(k)$. As an example, consider the case where we regulate the singularity by "hiding" it behind a horizon as in equation (19). In the limit $b \to 0$, we recover the background we are interested in (where the variables in Eqs.(19) and (26) are related by $z = \frac{R}{\omega}$). However, as we remarked in Section 3, for arbitrary values of b, the point $z = \frac{R}{b}$ corresponds to a conical singularity with deficit angle $\Delta\theta = 2\pi(1 - \frac{5}{2}\frac{R_0 b}{R^2})$. The deficit angle gives rise to a delta function singularity in the Einstein tensor, which can be interpreted as a brane (see also Refs. [25, 27]). In particular, if the field in question (gravity or, in this case, the scalar field) couples to this brane there could be a backreaction from the brane when a source is turned on elsewhere. One can imagine encoding the backreaction in some complicated form of boundary conditions at the brane (*e.g.* some linear combination of Dirichlet and Neumann boundary conditions with coefficients that could very well depend on the 4-momentum k). This would correspond to the freedom parameterized by $b(k)$ in Eq.(37) and could have important effects as we saw in Sections 2 and 3.

8 Appendix C:

In this Appendix, we derive the relation between the boundary-to-bulk propagator, relevant for the calculation of the two-point correlation function in the AdS/CFT correspondence [4, 5] and

the Neumann propagator, relevant to determine the gravitational potential (see also Ref. [11]). As usual, we restrict ourselves to the case of a scalar field. The calculation of the two point correlator proceeds by solving the bulk field equations with a specified value for the field at the boundary, *i.e.* by imposing Dirichlet (D) boundary conditions. The propagator appropriate for this kind of boundary conditions satisfies $G_D(k; z = \epsilon, z') = 0$. Then Eq.(30) (with $J(k, z) = 0$) gives the desired solution:

$$\phi(k, z') = \sqrt{g(z)}\, g^{zz}(z)\phi_0(k)\partial_z G_D(k; z, z')\Big|_{z=\epsilon} , \tag{38}$$

where $\phi(k, z' = \epsilon) = \phi_0(k)$. The boundary-to-bulk propagator is just

$$K(k; z') = \sqrt{g(z)}\, g^{zz}(z)\partial_z G_D(k; z, z')\Big|_{z=\epsilon} , \tag{39}$$

and, from Eq.(38) evaluated at $z' = \epsilon$, it satisfies

$$K(k; z' = \epsilon) = 1. \tag{40}$$

The two-point CFT correlation function can then be obtained via [4, 5]

$$A(k^2) \equiv \int dx e^{ikx} \langle \mathcal{O}(x)\mathcal{O}(0)\rangle = \sqrt{g(z)}\, g^{zz}(z)\partial_z K(k; z)\Big|_{z=\epsilon} . \tag{41}$$

Now note that the Green formula Eq.(30) also allows us to write the same solution for $\phi(k, z')$ in terms of the Neumann propagator if we specify the "correct" normal derivative, $\partial_z\phi(k, z)|_{z=\epsilon}$, at the boundary:

$$\phi(k, z') = -\sqrt{g(z)}\, g^{zz}(z)G_N(k; z, z')\partial_z\phi(k, z)\Big|_{z=\epsilon} \tag{42}$$

where

$$\partial_z\phi(k, z)\Big|_{z=\epsilon} = \phi_0(k)\partial_z K(k; z)\Big|_{z=\epsilon} \tag{43}$$

is obtained by differentiating Eq.(38) (we used the definition (39) of K). With this boundary condition, the solution (42) must be the same as solution (38) (at least in the Euclidean case). Thus, using the AdS/CFT relation (41), we find

$$K(k; z') = -G_N(k; \epsilon, z')A(k^2) \tag{44}$$

and setting $z' = \epsilon$ we obtain the final relation

$$G_N(k; \epsilon, \epsilon) = -A(k^2)^{-1}. \tag{45}$$

Note that this relation remains valid if we replace ϵ by an arbitrary z (also in the definition of A, (41)).

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